Symbolic SPICE

Symbolic SPICE®

Circuit Analyzer and Approximator

Application Note

AN-001:

Series Resonant Circuit

by Gregory M. Wierzba

Rev 102610

A) Introduction

The schematic shown below in Fig. 1 is a series *RLC* circuit. This simple circuit is capable of acting as a low-pass, high-pass, band-pass and band-stop filter [1].

A PSpice input file, **`rlc.cir`**, is given in Table 1. The same file is also accepted by Symbolic SPICE®.

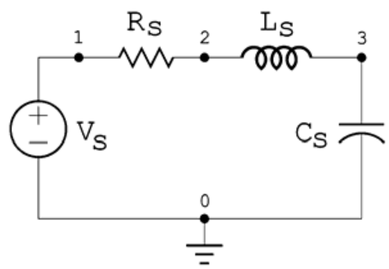

Figure 1. Series *RLC* circuit

Table 1. PSpice/Symbolic SPICE Input File

```
SERIES RLC CIRCUIT
VS 1 0 AC 1
RS 1 2 470
LS 2 3 1M
CS 3 0 .01U
.AC DEC 500 5K .5MEG
.PROBE
.END
```

B) Numerical Program Results

Running PSpice, we have the following numerical results with the element nodes indicated that create low-pass, high-pass, band-pass and band-stop filters:

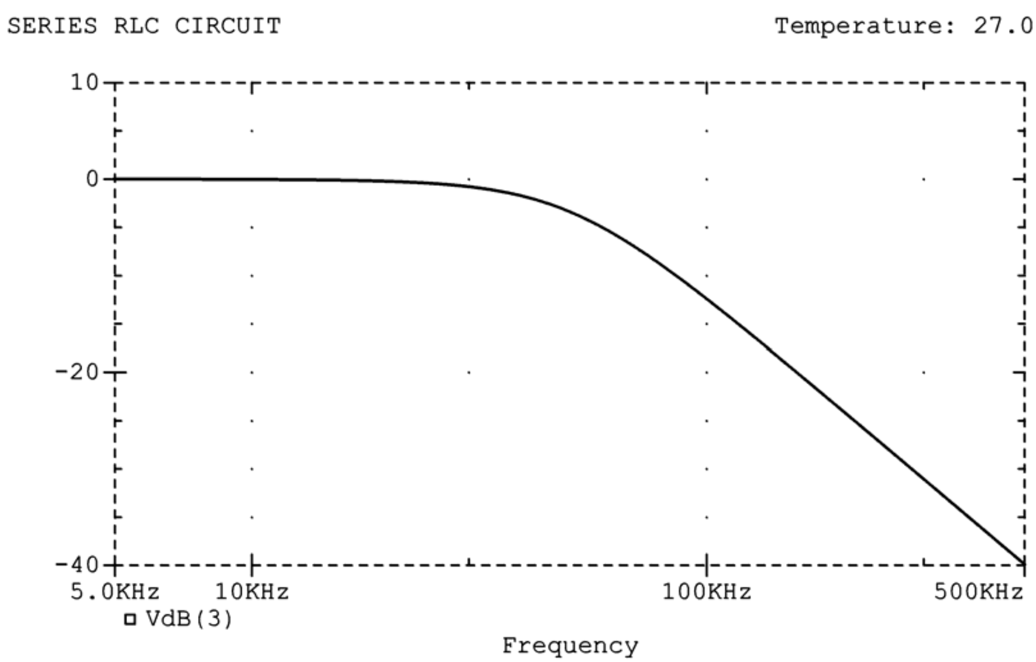

Figure 2. Low-pass response

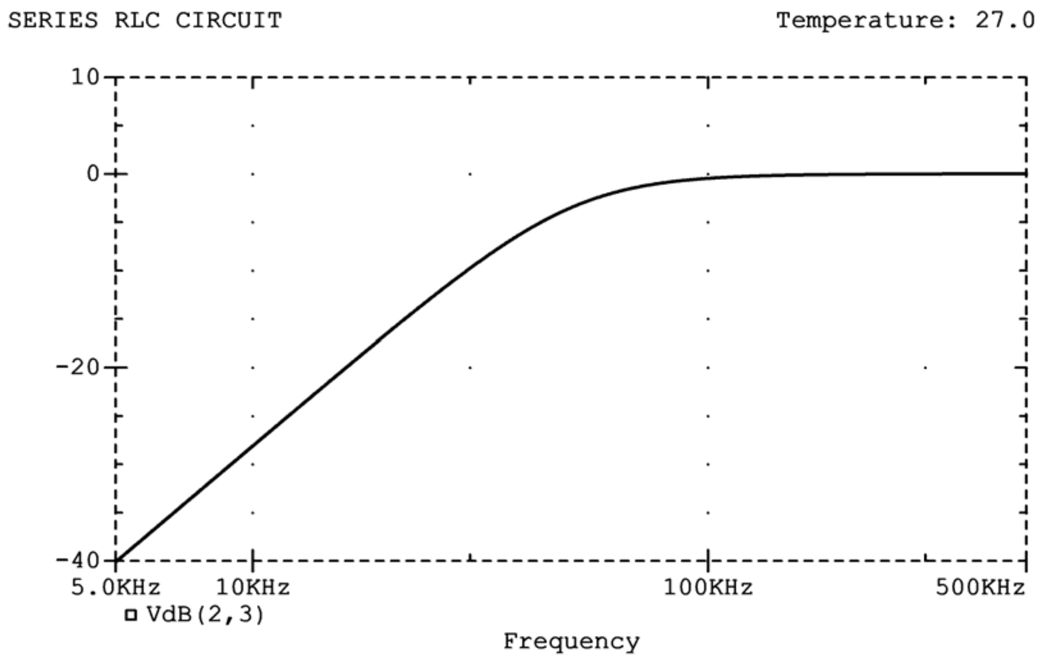

Figure 3. High-pass response

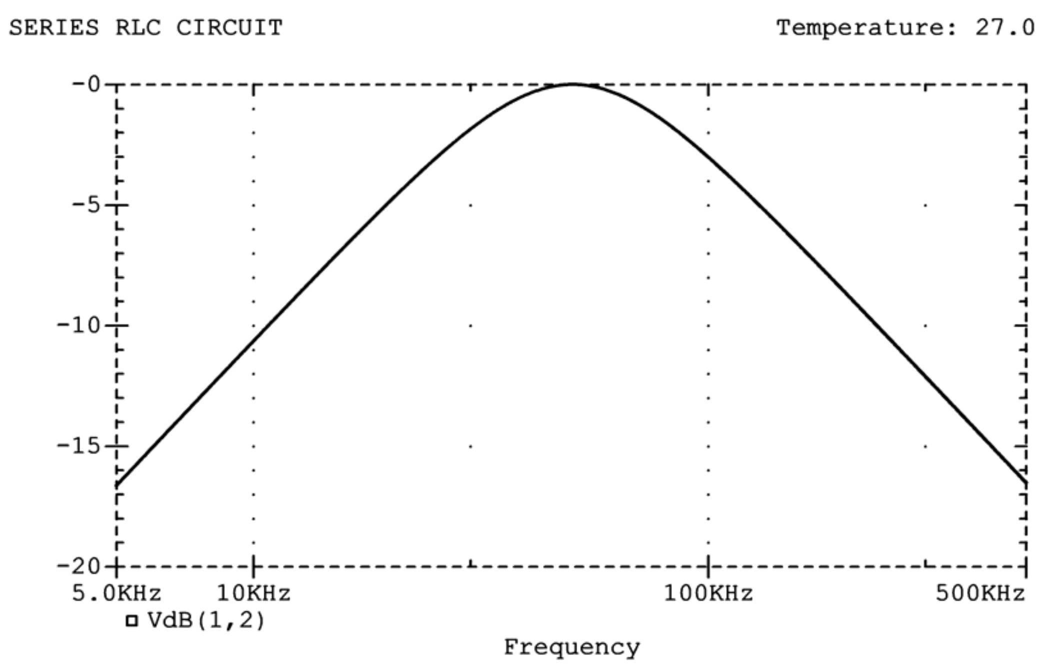

Figure 4. Band-pass response

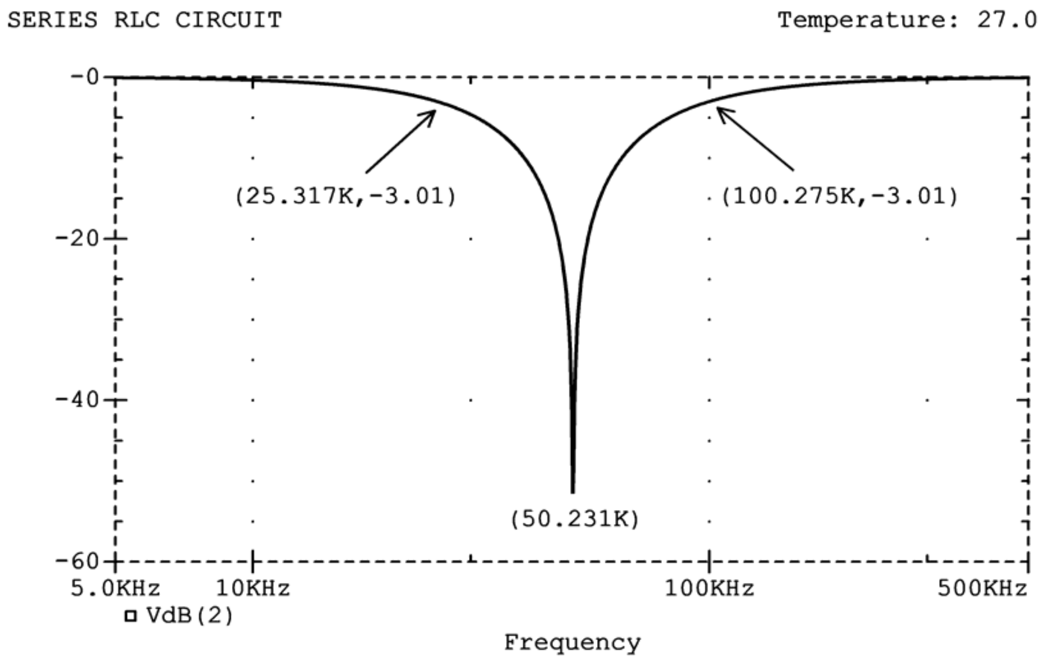

Figure 5. Band-stop response

C) Running Symbolic SPICE

Although a numerical program can show you what a circuit is capable of doing, Symbolic SPICE can show you why. Symbolic SPICE can also provide design equations. Running [2] the same input file shown in Table 1, the following are the prompts. The user responses are shown in bold:

```
Symbolic SPICE - Circuit Analyzer and Approximator
Demo Version 3.1
(C) Copyright 2010 by Willow Electronics, Inc.
```

INPUT FILE NAME [.cir] : **rlc**

OUTPUT FILENAME [rlc.det] : **(hit enter)**

Determinant string sorted according to orders of some variable? (y/n) : **n**

Numerical evaluation of the results? (y/n) : **y**

Discard terms if their magnitude falls below a threshold? (y/n) : **n**

Check and solve for second order filter functions? (y/n) : **y**

FILTER FUNCTION FILE NAME [rlc.fun] : **(hit enter)**

Solve for a variable or expression? (y/n) : **y**

Available Unknowns:


```
V1 V2 V3 V5
*Ignore nodes 4 and higher if present. They are used for internal numbering.
Valid Operators: +, -, *, /, ( ), { }, [ ]
Equation: v3
Solve for another variable or expression? (y/n) : y
Equation: v(2,3)
Solve for another variable or expression? (y/n) : y
Equation: v(1,2)
Solve for another variable or expression? (y/n) : y
Equation: v(2)
Solve for another variable or expression? (y/n) : n
```

D) Symbolic SPICE Determinant Output

The output file **`rlc.det`** listed in Table 2 is the matrix written by Symbolic SPICE and the transfer functions requested. The program formulates this matrix by using admittances [3].

For example, the resistance R_S is modeled as a conductance G_S with a value of 1/470 = 2.1276 ℧, the capacitance C_S is modeled as an admittance sC_S with a value of (0.01µ)s and the inductance L_S is modeled as an admittance 1/(sL_S) with a value of 1/[(1m)s] where s is the Laplace operator. All symbols are echoed as uppercase letters except for the Laplace operator.

Table 2. Symbolic SPICE Output File **`rlc.det`**

```
SERIES RLC CIRCUIT

[0  ] [-GS            GS-1           0              1              ][V1 ]
[0  ]=[0              -sLS           sCS+1          sLS-1          ][V2 ]
[1  ] [1              0              0              0              ][V3 ]
[0  ] [0              sLS+1          -1             -sLS           ][V5 ]

*Ignore nodes 4 and higher if present. They are used for internal numbering.

Numerator of: v3

TERMS SORTED ACCORDING TO POWERS OF s

s**0 terms:

 + GS

************************************************

NUMERICAL VALUE OF ABOVE SYMBOLIC RESULT

+ 0.00212766 * s**0

************************************************
```

```
Denominator of: v3

TERMS SORTED ACCORDING TO POWERS OF s

s**2 terms:

 + sLS*sCS*GS

s**1 terms:

 + sCS

s**0 terms:

 + GS

************************************************

NUMERICAL VALUE OF ABOVE SYMBOLIC RESULT

+ 2.12766e-014 * s**2 + 1e-008 * s**1 + 0.00212766 * s**0

************************************************

Numerator of: v(2,3)

TERMS SORTED ACCORDING TO POWERS OF s

s**2 terms:

 + sLS*sCS*GS

************************************************

NUMERICAL VALUE OF ABOVE SYMBOLIC RESULT

+ 2.12766e-014 * s**2

************************************************

Denominator of: v(2,3)

TERMS SORTED ACCORDING TO POWERS OF s

s**2 terms:

 + sLS*sCS*GS

s**1 terms:

 + sCS

s**0 terms:

 + GS

************************************************

NUMERICAL VALUE OF ABOVE SYMBOLIC RESULT

+ 2.12766e-014 * s**2 + 1e-008 * s**1 + 0.00212766 * s**0

************************************************
```

```
Numerator of: v(1,2)

TERMS SORTED ACCORDING TO POWERS OF s

s**1 terms:

 + sCS

************************************************

NUMERICAL VALUE OF ABOVE SYMBOLIC RESULT

+ 1e-008 * s**1

************************************************

Denominator of: v(1,2)

TERMS SORTED ACCORDING TO POWERS OF s

s**2 terms:

 + sLS*sCS*GS

s**1 terms:

 + sCS

s**0 terms:

 + GS

************************************************

NUMERICAL VALUE OF ABOVE SYMBOLIC RESULT

+ 2.12766e-014 * s**2 + 1e-008 * s**1 + 0.00212766 * s**0

************************************************

Numerator of: v(2)

TERMS SORTED ACCORDING TO POWERS OF s

s**2 terms:

 + sLS*sCS*GS

s**0 terms:

 + GS

************************************************

NUMERICAL VALUE OF ABOVE SYMBOLIC RESULT

+ 2.12766e-014 * s**2 + 0.00212766 * s**0

************************************************
```



```
Denominator of: v(2)

TERMS SORTED ACCORDING TO POWERS OF s

s**2 terms:

 + sLS*sCS*GS

s**1 terms:

 + sCS

s**0 terms:

 + GS

************************************************

NUMERICAL VALUE OF ABOVE SYMBOLIC RESULT

+ 2.12766e-014 * s**2 + 1e-008 * s**1 + 0.00212766 * s**0

************************************************
```

Symbolic SPICE's format is a collection of strings of symbols sorted by the (default) Laplace operator s. This is usually not how most people view equations, so you may need to do some minor editing. For example, the last transfer function **v(2)** is actually $V_2 / V_S = V_2 / V_1 = V_2$ since $V_1 = 1$. Thus we have

$$\frac{V_2}{V_1} = \frac{sL_S sC_S G_S + G_S}{sL_S sC_S G_S + sC_S + G_S} = \frac{s^2\, L_S C_S G_S + G_S}{s^2\, L_S C_S G_S + s\, C_S + G_S} = \frac{s^2 + \frac{1}{L_S C_S}}{s^2 + s\,\frac{1}{L_S G_S} + \frac{1}{L_S C_S}}$$

$$= \frac{s^2 + \frac{1}{L_S C_S}}{s^2 + s\,\frac{R_S}{L_S} + \frac{1}{L_S C_S}}$$

Likewise for the numerical results,

$$\frac{V_2}{V_1} = \frac{2.12766e^{-14}\, s^2 + 0.00212766}{2.12766e^{-14}\, s^2 + 1e^{-8}\, s + 0.00212766} = \frac{s^2 + 100\text{G}}{s^2 + 470\text{k}\, s + 100\text{G}}$$

E) Symbolic SPICE Filter Function Output

Symbolic SPICE will examine any transfer function to determine if its denominator is second order in the Laplace operator s. If this is the case then it finds the parameters associated with the different filter functions. Symbolic SPICE finds formulas for Q_o, ω_o and H_i where some of the various cases of i are listed on the following page:

Low-pass filter

$$T_{lp} = \frac{H_{lp}\,\omega_o^2}{s^2 + s\,\frac{\omega_o}{Q_o} + \omega_o^2}$$

High-pass filter

$$T_{hp} = \frac{H_{hp}\,s^2}{s^2 + s\,\frac{\omega_o}{Q_o} + \omega_o^2}$$

Band-pass filter

$$T_{bp} = \frac{H_{bp}\,\frac{\omega_o}{Q_o}\,s}{s^2 + s\,\frac{\omega_o}{Q_o} + \omega_o^2}$$

Band-stop filter

$$T_{bs} = \frac{H_{bs}\,(s^2 + \omega_z^2)}{s^2 + s\,\frac{\omega_o}{Q_o} + \omega_o^2}$$

The output file **`rlc.fun`** listed in Table 3 contains the filter parameters solved for by Symbolic SPICE.

Table 3. Symbolic SPICE Output File **`rlc.fun`**

```
SERIES RLC CIRCUIT

SECOND ORDER FILTER PARAMETERS:

Qo is:

( + GS)*SQRT{ + LS}
-------------------
SQRT{ + CS}

 = 0.672825

Wo**2 is:

( + 1)
--------
( + LS*CS)
```

```
fo = 50329.2Hz

***************************************

There exists a LOW PASS filter for : v3

LOW PASS GAIN (Hlp) is:

( + 1)
--------
 + 1

 =      1

***************************************

There exists a HIGH PASS filter for : v(2,3)

HIGH PASS GAIN (Hhp) is:

( + 1)
--------
 + 1

 =      1

***************************************

There exists a BAND PASS filter for : v(1,2)

BAND PASS GAIN (Hbp) is:

( + 1)
--------
 + 1

 =      1

***************************************

There exists a BAND STOP filter for : v(2)

BAND STOP GAIN (Hbs) is:

( + 1)
--------
 + 1

 =      1

BAND STOP FREQUENCY (Wz**2) is:

( + 1)
--------
( + LS*CS)

fz = 50329.2Hz

***************************************
```


F) Comparison of Results

From Fig. 5, we can read the numerical value of $f_o = f_z$ = 50.231k Hz. The exact value of f_o is 50.3292k Hz. This is a difference of 0.195% which is approximately the default accuracy of PSpice. Likewise we can calculate the numerical value of Q_o from the data [1] in Fig. 5 where $Q_o = f_o$ /(-3dB bandwidth) = 50.231k / (100.275k - 25.317k) = 0.67012. The exact value is 0.672825. This is a difference of 0.402%.

Besides the numerical results, we now have the formulas for

$$f_o = f_z = \frac{1}{(2\pi)\sqrt{L_S C_S}}$$

and

$$Q_o = G_S\sqrt{\frac{L_S}{C_S}} = \frac{1}{R_S}\sqrt{\frac{L_S}{C_S}}$$

To turn these formulas into a design procedure[1]:

Pick L_S = Standard inductor value

then

$$C_S = \frac{1}{(2\pi f_o)^2 L_S}$$

and

$$R_S = \frac{1}{Q_o}\sqrt{\frac{L_S}{C_S}}$$

G) Conclusion

Symbolic SPICE can be used to analyze passive filters. Symbolic SPICE can recognize second order transfer functions in the Laplace operator s. It can also determine the filter type such as low-pass, high-pass, band-pass and band-stop and then solve for the filter parameters ω_o, Q_o and H_i.

H) References

[1] G. M. Wierzba, *ECE 202: Electric Circuits and Systems II - Class Notes,* Ch.12, pp. 26-42.This e-book is available at http://stores.lulu.com/willowepublishing

[2] G. M. Wierzba, *Symbolic SPICE User Manual*, p. 6. This e-book is available at http://stores.lulu.com/willowepublishing

[3] G. M. Wierzba, *Symbolic SPICE User Manual*, p. 7-8.

